# **Lesson Objectives**

1. Simplify a square root – Perfect Square method
2. Simplify a square root – Pairs and Spares method
3. Simplify square roots containing variables

# **Simplify a Square Root – Perfect Square Method**

## Review of **Perfect Squares**

A **perfect square** is a number that has two \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ factors.

To simplify square roots, it’s really helpful if you know at least the first 15 perfect squares:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

## Simplify a Square Root – **Perfect Square Method**

**Radicand** – the value or amount \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the root

**Index** – the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of root you are taking

With a **square root**, the index of **2** is not written – it is omitted.

A **square root** is considered **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** if:

the **radicand contains \_\_\_\_\_\_ perfect square factors**.

* **STEP 1.** Inside the square root, divide the radicand into two factors:
  + the **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ perfect square** that divides into the radicand
  + its “\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_” factor that goes with it
* **STEP 2.** Each of those factors gets its \_\_\_\_\_\_\_\_\_ square root, multiplied together.
* **STEP 3.** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the perfect square root into its whole number.
* **STEP 4.** Leave the “buddy” factor \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the square root as the remaining reduced radicand.
* **EXAMPLE:** Simplify by factoring out the largest perfect square. [R.7.37]
* **STEP 1.** Inside the square root, divide the radicand into two factors:
  + the **largest perfect square** that divides into the radicand
  + its “buddy” factor that goes with it

To find the **largest perfect square factor** of 192, you need to:

* + Test the perfect squares by \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 192 by each perfect square
  + No \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, no remainder
  + You only need to test perfect squares to about \_\_\_\_\_\_\_\_\_\_-way to 192, or 96

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | 100 is more than half-way |

* **STEP 1.** Rewrite as

64 is the largest perfect square factor of 192

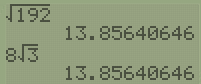
Its “buddy” factor is 3 because

* **STEP 2.** Each of those factors gets its own square root, multiplied together.
* **STEP 3.** Simplify the perfect square root into its whole number.
* **STEP 4.** Leave the “buddy” factor inside the square root as the remaining reduced radicand.

**ANSWER:**

You can easily verify that on your calculator.

Just verify the approximate decimal equivalents:



* **EXAMPLE:** Simplify. [\*Angel 11.3.11]

448 has several perfect square factors, but we want the largest one.

|  |  |
| --- | --- |
|  | 4 is not the largest perfect square factor because the remaining factor, \_\_\_\_\_, still divides down by at least the perfect square \_\_\_\_. |
|  | 16 is not the largest perfect square factor because the remaining factor, \_\_\_\_\_, still divides down by the perfect square \_\_\_\_. |
|  | 64 is the largest perfect square factor because the remaining factor, \_\_\_\_\_, does \_\_\_\_\_\_\_\_\_ divide down by any perfect squares. |

* **STEP 1.** Rewrite as

64 is the largest perfect square factor of 448

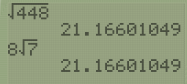
Its “buddy” factor is 7 because

* **STEP 2.** Each of those factors gets its own square root, multiplied together.
* **STEP 3.** Simplify the perfect square root into its whole number.
* **STEP 4.** Leave the “buddy” factor inside the square root as the remaining reduced radicand.

**ANSWER:**

You can easily verify that on your calculator.

Just verify the approximate decimal equivalents:



Caution: Don’t be too over-reliant upon the calculator!

For example, also equals ; however, is not simplified because the radicand \_\_\_\_\_\_ still divides down by a perfect square, \_\_\_\_\_.

# **Simplify a Square Root – “Pairs and Spares” Method**

The challenge with the **Perfect Square** method is that sometimes it’s \_\_\_\_\_\_\_\_\_\_\_\_\_\_ to determine the largest perfect square factor because the radicand is either large or otherwise unfamiliar, or it may not simplify at all.

An alternate, sometimes \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and more \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ method is called the “Pairs and Spares” method, which utilizes a technique involving a **factor tree**, or the **prime factorization**.

## **Prime Factorization** – make a **Factor Tree**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** number: a whole number whose **only factors are 1 and itself**.

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** number: a whole number that is **NOT prime**; it is **composed** of **prime factors**. It has additional factors besides 1 and itself.

Note that the number 1 is neither prime nor composite.

**Prime factorization:** an arrangement of **\_\_\_\_\_\_\_\_\_\_\_\_ factors** whose product is a given number. EVERY whole number (greater than 1) has a UNIQUE prime factorization.

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_:** a systematic way to divide down a whole number into its unique prime factors, or is **prime factorization**.

**STEP 1. “\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_-\_\_\_\_\_\_\_\_\_\_”** the given number into 2 factors.

* If there’s more than one way to have 2 factors, then you can simply choose whichever you prefer – it doesn’t matter.

**STEP 2.** If either of the 2 factors is **prime**, then **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** it.

**STEP 3.** If either of the 2 factors is **composite**, then **“branch-off”** of that number into 2 factors as well.

**STEP 4.** (If needed) Continue the process until your factor tree has \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ left but a collection of circled prime numbers.

**STEP 5. Write the prime factorization**:

* List all of the circled numbers together,
* Separated with a multiplication sign in between each factor
* (EXAMPLE): Use a factor tree to find the prime factorization of 405.

**ANSWER:** The prime factorization of 405 is:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Simplify a Square Root –“**Pairs and Spares”** Method

**STEP 1.** Get the **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** of the radicand using a **factor tree**.

**STEP 2.** Write the PF as the updated radicand \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the square root.

**STEP 3A.** **Circle** any **\_\_\_\_\_\_\_\_\_\_\_\_** of identical factors; that is, a perfect square.

* Each *pair* of identical factors *inside* the square root simplifies to a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_** factor **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** the square root (to its LEFT).
* Do this for *each* identified pair of identical factors.

**STEP 3B.** **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** any remaining unpaired factors still in the radicand (*inside* the square root) – these are **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**.

**STEP 4. \_\_\_\_\_\_\_\_\_\_\_\_\_ together** either the *outside* factors or the *inside* factors, if needed.

* **EXAMPLE:** Simplify the expression. [\*Angel 11.3.19]

**STEP 1.** From the example above, the **prime factorization** of 405 is

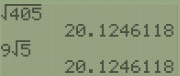
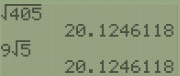
**STEP 2.** Update the **radicand**:

**STEP 3.** **Circle pairs**, **Underline spares.**

Each pair **simplifies** to a single:

**STEP 4. Multiply** ***outside*** factors :

**Multiply** ***inside*** factors: (not needed)

**ANSWER:** simplifies to 

# **Simplify a Square Root Containing Variables** (“Pairs and Spares” Method)

You can simplify expressions with variables by using the basic definition of an **\_\_\_\_\_\_\_\_\_\_\_\_\_**.

For example, you could write out the factors of as and then circle pairs similar to how you do with constants.

* **EXAMPLE:** Simplify by factoring. Assume that all expressions under radicals represent nonnegative numbers. [\*Blitzer 10.3.39]

**STEP 1 and 2. Prime factorization, update radicand.**

Rewrite in the radicand using definition of exponent:

**STEP 3. Circle pairs, underline spares.**

**Simplify to singles.**

**Multiply** outsides **together.**

**STEP 4. Multiply** spares together inside: (not needed)

**ANSWER:**  simplifies to

Notice when the exponent is very LARGE, this can be rather \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

There’s an easier way. Here’s the previous problem again, earlier in the problem:

How many **pairs** are there? \_\_\_\_\_\_ How many **spares** are there? \_\_\_\_\_

An **\_\_\_\_\_\_\_\_\_** exponent can always be written as the previous **\_\_\_\_\_\_\_\_\_\_\_\_\_** exponent and a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Examples: or or

Pairs & Spares: 8 pairs, 1 spare 5 pairs, 1 spare 3 pairs, 1 spare

Sq. Rt. Is Exponent/2:

Simplified

## **Simplify square roots containing both variables and constants**

* **EXAMPLE:** Express in simplified form. [R.7.47]

Assume that all variables represent positive real numbers.

**CONSTANT**

**STEP 1. Prime Factorization.**

**STEP 2. Update radicand.**

**STEP 3. Circle pairs, underline spares.**

**Simplify to singles.**

**STEP 4. Multiply *outside*** factors**:**

**Multiply *inside*** factors (not needed)

**VARIABLES**

Write as **separate square roots**:

Rewrite **odd** exponents as the previous even and a spare:

**Simplify** even exponents by dividing by 2:

**MERGE**

Merge together the answer portions from the **constants** and the **variables**.

CONSTANTS: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ VARIABLES:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**MERGED – FINAL ANSWER:** simplifies to \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sources Used:

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